PDT-II: Mass Hierarchy and Sector Amplification

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Abstract

PDT-II develops the structural mass hierarchy implied by Phase Transport Geometry (PTG) and the finite coherence capacity introduced in PDT-I. The PTG mass functional

$$m_i = m_0 \, C_i W_i L_i \, g_i^2 \, S_i \, T_i$$

contains contributions from closure, winding, loading, excitation mode, sector symmetry, and return-closure invariants. PDT-II analyses how these factors combine to produce a three-tier structural hierarchy across the singlet, doublet, and triplet sectors. The return-closure invariant $T_3 = 9$, derived in PDT-MC-1, provides the dominant amplification of the triplet structural mass functional. The hierarchy developed in this paper is the structural hierarchy of stable PTG defects; its relationship to the observed Standard Model particle spectrum is explored in later work.

This paper is part of the Phase Transport Geometry / Phase Differential Theory series, which develops a unified geometric and operator-theoretic framework for relational transport.

1 Introduction

PTG-5 established the structural mass functional

$$m_i = m_0 \, C_i W_i L_i \, g_i^2 \, S_i \, T_i.$$

PDT-II develops one of its principal consequences: the emergence of a structured mass hierarchy across the three closure sectors. The hierarchy developed here is the structural hierarchy of stable PTG defects rather than the complete Standard Model particle spectrum. Connections to observed particle masses are explored in later work.

The key structural feature is the amplification of the triplet sector by the return-closure invariant

$$T_3 = 9.$$

2 Sector Structure and Baseline Factors

PTG-4 decomposes the phase manifold into:

triplet sector (3D), doublet sector (2D), singlet sector (1D).

Each sector contributes different baseline factors to the mass functional.

Baseline Factors

The baseline factor is

$$B_i = C_i W_i L_i.$$

The minimal nontrivial values are:

$$W_{\min} = 9, \qquad L_{\min} = 4.$$

Thus:

$$B_1 = 36, \qquad B_2 = 72, \qquad B_3 = 108.$$

3 Excitation Modes

PDT-MC-1 shows that only three excitation modes are stable:

$$g = 1, 2, 3.$$

The excitation contribution is quadratic:

$$g^2.$$

4 Sector Symmetry Factors

The symmetry factors arise from the structural algebras:

$$A(1) = \mathbb{C}, \qquad A(2) = \mathrm{End}(\mathbb{C}^2), \qquad A(3) = \mathrm{End}(\mathbb{C}^3).$$

Interpretation of S_i

The factors S_1, S_2, S_3 encode the degeneracy of the defect under the structural algebra of its sector. They count the number of independent symmetry-preserving excitations available to the defect. These factors do not represent physical gauge symmetries; they arise purely from the closure-induced sector structure of PTG.

5 Return-Closure Invariants

Under finite coherence capacity (PDT-I), residual phase cannot persist. PDT-MC-1 shows that the return algebra of the triplet sector is

$$A(3) = \mathrm{End}(\mathbb{C}^3),$$

which has dimension

$$T_3 = 9.$$

Thus:

$$T_1 = 1, \qquad T_2 = 1, \qquad T_3 = 9.$$

Sector Amplification Theorem. Under finite coherence capacity and return-closure, the triplet sector acquires the invariant

$$T_3 = \dim \mathrm{End}(\mathbb{C}^3) = 9,$$

which provides the dominant amplification of the triplet structural mass functional.

6 Multiplicative Structure of the Mass Functional

The structural mass functional is multiplicative because each factor represents an independent geometric or algebraic contribution to the coherence burden:

- closure constraints (C_i),
- winding (W_i),
- curvature loading (L_i),
- excitation mode (g_i^2),
- sector symmetry (S_i),
- return-closure invariants (T_i).

No factor can be absorbed into another without losing structural information. The multiplicative structure is therefore the natural and minimal way to combine these independent contributions.

7 Mass Hierarchy

Combining all contributions yields:

$$m_{\text{singlet}} = m_0\, 36\, g^2 S_1,$$

$$m_{\text{doublet}} = m_0\, 72\, g^2 S_2,$$

$$m_{\text{triplet}} = m_0\, 108 \times 9\, g^2 S_3 = m_0\, 972\, g^2 S_3.$$

Thus the ratios are:

$$\frac{m_{\text{doublet}}}{m_{\text{singlet}}} = 2\, \frac{g^2 S_2}{g^2 S_1}, \qquad \frac{m_{\text{triplet}}}{m_{\text{doublet}}} = 13.5\, \frac{g^2 S_3}{g^2 S_2}.$$

Within the PTG/PDT framework, the triplet sector carries a larger structural coherence burden owing to the return-closure invariant $T_3 = 9$, leading to greater mass in the associated structural mass functional.

8 Interpretation

The structural mass hierarchy arises from:

- closure structure,
- winding and loading,
- excitation modes,
- sector symmetry,
- return-closure invariants.

The down-sector invariant $T_3 = 9$ is the dominant amplification factor.

9 Calibration vs Physical Comparison

In the present formulation, the electron serves as the reference low-energy stable charged defect. This calibrates the primitive mass scale m_0:

$$\frac{m_e}{m_0} = 72.$$

This calibration is conceptually distinct from later comparison with observed particle masses. The present paper develops the structural hierarchy of PTG defects; detailed Standard Model comparisons are deferred to later work.

10 Conclusion

PDT-II develops the structural mass hierarchy implied by PTG and PDT-I. The key results are:

- the PTG mass functional produces a structured hierarchy,
- the down-sector invariant amplifies triplet masses,
- the hierarchy exhibits strong separation between sectors.

This hierarchy forms the structural foundation for the collapse dynamics developed in PDT-III.