PDT-III: Differential Transport, Collapse Dynamics, and the Coherence Horizon

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Abstract

PDT-III develops the dynamical consequences of finite coherence capacity (Axiom 0) introduced in PDT-I. The compact transport potential of PTG-2 induces a gradient-flow dynamical system on the minimal phase manifold $(S^1)^6$, and the curvature operator generates a loading accumulation that grows along the flow. When the total loading exceeds the coherence capacity $C_{\max}$, the system undergoes coherence collapse. PDT-III formalises this behaviour, defines the coherence horizon, and shows that the collapse time is a stopping time determined entirely by the transport dynamics together with finite coherence capacity. Existence and uniqueness of solutions to the differential transport system are established in PDT-MC-1. PDT-III provides the initial physical interpretation of the PTG framework and the dynamical foundation for the mass hierarchy and collapse behaviour developed in PDT-II and PDT-III.

This paper is part of the Phase Transport Geometry / Phase Differential Theory series, which develops a unified geometric and operator-theoretic framework for relational transport.

1 Introduction

PTG-1 through PTG-5 established the geometric and algebraic structure of Phase Transport Geometry. PDT-I introduced Axiom 0, asserting that coherent physical systems have a finite coherence capacity. PDT-II developed the structural mass hierarchy implied by the PTG mass functional.

PDT-III now develops the dynamical system implied by compact transport and finite coherence capacity. The key ideas are:

- transport is modelled as a gradient flow,
- curvature loading accumulates along trajectories,
- collapse occurs when loading exceeds capacity,
- the coherence horizon bounds coherent evolution.

2 Differential Transport System

Let $y(t) \in (S^1)^6$ denote the phase configuration at time t. The compact transport potential is

$$T(\theta) = K(1 - \cos\theta).$$

Gradient Flow

The transport dynamics are modelled as the gradient flow of the compact transport potential:

$$\frac{dy}{dt} = -\nabla T(y(t)).$$

Why gradient flow?

Gradient flow is the canonical transport law for compact relational phase geometry because:

- it ensures evolution proceeds along steepest descent of the compact potential,
- it guarantees bounded trajectories on the compact manifold $(S^1)^6$,
- it is compatible with the closure-compatible metric of PTG-3,
- it preserves the periodic structure of the phase coordinates,
- it yields a well-posed dynamical system with unique solutions (PDT-MC-1).

3 Curvature Loading

The curvature operator is the geometric Hessian

$$K(y)X = \nabla_X(\nabla T)(y).$$

The total loading is defined as

$$L(t) = \mathrm{Tr}(K(y(t))).$$

Loading Accumulation Theorem

Assumptions:

- the transport potential T is smooth and periodic (PTG-2),
- the curvature operator K is smooth on $(S^1)^6$,
- the metric is closure-compatible and positive on the reduced space (PTG-3),
- the gradient flow is well-defined and unique (PDT-MC-1).

Theorem. Along every transport trajectory,

$$\frac{dL}{dt} = \mathrm{Tr}(K(y(t))),$$

so whenever $\mathrm{Tr}(K)$ remains positive, the accumulated loading $L(t)$ increases monotonically.

This expresses the fact that curvature loading accumulates continuously along the gradient-flow trajectory.

4 Collapse Criterion

Axiom 0 states that collapse occurs when

$$C(t) = \mathrm{Tr}(K(y(t))) > C_{\max}.$$

Define the collapse time

$$t_{\mathrm{col}} = \inf\{t > 0 : C(t) > C_{\max}\}.$$

The collapse time is a stopping time determined entirely by the transport dynamics together with the finite coherence capacity.

5 Coherence Horizon

The coherence horizon is the maximal time interval on which the system remains coherent:

$$0 \le t < t_{\mathrm{col}}.$$

Coherence Horizon Theorem. If compact transport and finite coherence capacity hold, then coherent evolution is defined only on the interval

$$0 \le t < t_{\mathrm{col}}.$$

Beyond t_{col}, the configuration necessarily transitions to a new coherent state. PDT-III deliberately does not specify the post-collapse dynamics; these are deferred to future work.

6 Interpretation

Within the PTG/PDT framework:

- curvature loading accumulates along gradient-flow trajectories,
- finite coherence capacity forces collapse,
- return-closure is required for stability,
- defects with larger structural coherence burden accumulate loading more rapidly.

These ideas underpin the mass hierarchy (PDT-II) and the collapse dynamics developed in later work.

7 Discussion

PDT-III provides the dynamical foundation for the PTG/PDT programme. The key insights are:

- compact transport induces curvature loading,
- loading accumulates monotonically along trajectories,
- finite coherence capacity forces collapse,

- the collapse time is a stopping time,
- the coherence horizon bounds coherent evolution.

These results complete the initial physical interpretation of PTG and prepare the ground for the mathematical consolidation in PDT-MC-1.

8 Conclusion

PDT-III develops the collapse dynamics implied by compact transport and finite coherence capacity. The coherence horizon provides a natural dynamical limit, and the collapse criterion explains the stability and mass hierarchy of defects. Combined with PDT-I and PDT-II, this paper completes the initial physical interpretation of Phase Transport Geometry.