# PTG-1: Closure Architecture and the Minimal Phase Manifold


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**Abstract**

Phase Transport Geometry (PTG) begins from a primitive structural axiom: the existence of an irreducibly collective triplet closure relation. This axiom generates the closure hierarchy (1,2,3) and determines the minimal compact phase manifold capable of supporting independent singlet, doublet, and triplet sectors. Under explicit structural assumptions, we show that this manifold is diffeomorphic to $(S^1)^6$ and prove its conditional minimality and uniqueness. PTG-1 provides the geometric foundation for compact transport (PTG-2), the closure-compatible metric (PTG-3), sector structure (PTG-4), defect mass scaling (PTG-5), and the physical interpretation developed in the PDT series.

This paper is part of the Phase Transport Geometry / Phase Differential Theory series, which develops a unified geometric and operator-theoretic framework for relational transport.


## 1 Introduction

Phase Transport Geometry is built on the idea that coherent physical systems satisfy collective neutrality conditions among phase degrees of freedom. Rather than beginning with fields or symmetries, PTG begins with closure. The primitive closure axiom is the triplet relation

$$1 + \omega + \omega^2 = 0, \qquad \omega = e^{2\pi i/3},$$

which is taken as a structural axiom rather than a derived result. Every physical theory begins with primitive assumptions; PTG makes this one explicit.

From this axiom, PTG develops:

- the closure hierarchy (1,2,3),
- compact transport and curvature loading (PTG-2),
- the closure-compatible metric (PTG-3),
- sector structure and structural algebras (PTG-4),
- defect mass scaling (PTG-5),
- the physical interpretation (PDT-I, PDT-II, PDT-III),
- the mathematical closure framework (PDT-MC-1).

This paper establishes the closure architecture and the minimal phase manifold.

# 2 The Primitive Closure Axiom

### Axiom C3 (Triplet Closure)

The primitive closure operation is the irreducibly collective relation

$$1 + \omega + \omega^2 = 0.$$

### Interpretation

- It is the smallest non-binary neutral composite.
- It requires three independent phase degrees of freedom.
- It motivates the existence of a triplet sector.

# 3 The Closure Hierarchy (1,2,3)

Closure can occur at three minimal orders:

- **Singlet (1):** A single phase with integer winding.
- **Doublet (2):** A pair of phases related by orientation (chirality).
- **Triplet (3):** The primitive closure axiom above.

### Internal completeness of the hierarchy

Given the primitive closure axioms, the chain

$$1 - 2 - 3 \;\Rightarrow\; (S^1)^6$$

is internally complete within the PTG framework. No additional closure orders are required to support the sector structure used throughout the PTG/PDT series.

# 4 The Minimal Phase Manifold

Each closure sector contributes a number of independent phase degrees of freedom:

$$\text{triplet: } 3, \quad \text{doublet: } 2, \quad \text{singlet: } 1.$$

### Conditional Minimality and Uniqueness

Assume:

(a) the primitive closure hierarchy consists of independent singlet, doublet, and triplet sectors;

(b) these sectors require respectively 1, 2, and 3 independent periodic phase coordinates;

(c) the phase coordinates factor globally as independent compact circles.

Then the minimal compact phase manifold supporting the closure hierarchy is

$$M_{\text{PTG}} = (S^1)^6,$$

unique up to diffeomorphism and sector relabelling.

### Proof

Assumptions (a) and (b) require $1 + 2 + 3 = 6$ independent periodic coordinates. Assumption (c) forces each coordinate to be an $S^1$. Any compact manifold with six independent circle factors is diffeomorphic to $(S^1)^6$, up to relabelling. No lower-dimensional compact manifold can realise all three sectors simultaneously. □

## 5 Discussion

PTG-1 establishes:

- the primitive closure axiom,
- the closure hierarchy (1,2,3),
- the minimal phase manifold $(S^1)^6$,
- conditional minimality and uniqueness under explicit structural assumptions.

This structure is the geometric substrate for:

- compact transport geometry (PTG-2),
- the closure-compatible metric (PTG-3),
- sector structure and structural algebras (PTG-4),
- defect mass scaling (PTG-5),
- finite coherence capacity and the mass gap (PDT-I),
- the mass hierarchy (PDT-II),
- collapse dynamics (PDT-III),
- the mathematical closure framework (PDT-MC-1).

## 6 Conclusion

PTG-1 provides the geometric foundation of Phase Transport Geometry. The closure hierarchy determines the minimal phase manifold $(S^1)^6$, which supports the transport, metric, sector, and mass structures developed in later PTG and PDT papers.