# PDT-I: Axiom 0, Finite Coherence Capacity, and the Emergence of a Positive Stability Threshold


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**Abstract**

Phase Differential Theory (PDT) provides the physical interpretation of the geometric structures developed in Phase Transport Geometry (PTG). This first paper in the PDT series introduces Axiom 0, which asserts that coherent physical systems possess a finite coherence capacity. When the total curvature loading exceeds this capacity, coherence collapses and the system transitions to a new configuration. Combined with compact transport (PTG-2), the closure-compatible metric (PTG-3), and the sector structure (PTG-4), Axiom 0 implies the existence of a positive stability threshold that is interpreted as a positive mass gap within the PTG/PDT framework. PDT-I therefore establishes the physical foundation for the mass hierarchy developed in PDT-II and the collapse dynamics formalised in PDT-III.

This paper is part of the Phase Transport Geometry / Phase Differential Theory series, which develops a unified geometric and operator-theoretic framework for relational transport.


## 1 Introduction

PTG-1 through PTG-5 established the geometric and algebraic structure of Phase Transport Geometry:

- closure hierarchy and minimal manifold (PTG-1),
- compact transport and curvature loading (PTG-2),
- closure-compatible metric (PTG-3),
- sector structure and structural algebras (PTG-4),
- structural mass functional (PTG-5).

PDT provides the physical interpretation of these structures. PDT-I introduces Axiom 0, which asserts that coherent physical systems have a finite capacity to support curvature loading. This axiom is motivated by the compactness of the PTG phase manifold, the boundedness of the compact transport potential, and the finite distinguishability of physically accessible configurations.

Axiom 0 has three major consequences:

- it distinguishes local instability from global coherence collapse,
- it enforces return-closure for stable defects,
- it implies a positive stability threshold interpreted as a positive mass gap.

## 2 Axiom 0: Finite Coherence Capacity

### Axiom 0 (Finite Coherence Capacity)

Every coherent physical system has a finite coherence capacity $C_{\max}$. If the total curvature loading

$$C(t) = \mathrm{Tr}(K(y(t)))$$

exceeds $C_{\max}$, coherence collapses and the system transitions to a new configuration.

### Interpretation

The curvature operator $K$ introduced in PTG-2 measures the local and global stability of a configuration. The trace

$$C(t) = \mathrm{Tr}(K(y(t)))$$

measures the total coherence loading across all phase directions.

**Definition (Collapse).** Collapse refers to the loss of the present coherent configuration once the total curvature loading exceeds the coherence capacity. It does *not* imply wavefunction collapse; it is a structural transition to another admissible coherent configuration.

Axiom 0 asserts that:

- coherence cannot be sustained indefinitely,
- loading accumulates during transport,
- collapse occurs when loading exceeds capacity.

## 3 Local Instability vs Global Collapse

PTG-2 distinguished:

- local instability: governed by $\lambda_{\max}(K)$,
- global collapse: governed by $C(t) = \mathrm{Tr}(K)$.

A system may be locally stable (all eigenvalues below threshold) yet still collapse globally if the total loading becomes too large. This distinction is essential for:

- the positive stability threshold (this paper),
- the mass hierarchy (PDT-II),
- coherence collapse and the coherence horizon (PDT-III).

## 4 Return-Closure as a Physical Requirement

Under finite coherence capacity, residual phase cannot persist. A stable defect must therefore satisfy a return-closure condition:

$$T(y)v = v,$$

where $T(y)$ is the transport operator around a closed loop.

PDT-MC-1 shows that enforcing return-closure in the triplet sector introduces the down-sector invariant

$$T_3 = 9.$$

Axiom 0 provides the physical justification:

- residual phase contributes to curvature loading,
- loading accumulates under transport,
- finite capacity forces return-closure.

## 5 Derived Axiom 0: Structural Origin of Finite Coherence Capacity

**Finite Coherence Capacity Theorem.** Let $M_{\text{phys}}$ be the reduced PTG phase manifold obtained by quotienting closure-equivalent directions. Suppose:

1. $M_{\text{phys}}$ is compact;
2. transport is generated by a smooth compact periodic potential;
3. the closure-compatible metric is positive definite on the reduced quotient space;
4. physically accessible configurations are finitely distinguishable;
5. relational phase gradients are bounded.

Then every physically realisable coherent configuration has a finite coherence capacity. That is, there exists a finite constant

$$C_{\max} < \infty$$

such that coherent evolution is only physically admissible while

$$C(t) = \mathrm{Tr}(K(y(t))) \leq C_{\max}.$$

### Proof Sketch

PTG-1 establishes that the primitive phase manifold is compact:

$$M = (S^1)^6.$$

PTG-3 removes closure-redundant directions and gives a positive-definite metric on the reduced quotient space. Therefore physically distinct configurations are measured on $M_{\text{phys}}$.

PTG-2 introduces compact transport through a smooth periodic potential

$$T(\theta) = K(1 - \cos\theta),$$

whose derivatives are bounded on the compact manifold. Hence the associated curvature operator is bounded on the physically reduced space.

Finite distinguishability and bounded relational gradients imply that no physically realisable coherent state can carry unbounded curvature loading. Therefore the total admissible curvature loading is finite.

Thus finite coherence capacity is not an independent primitive postulate, but a derived structural consequence of compact relational phase geometry, finite distinguishability, bounded relational gradients, and closure-compatible transport.

### Reformulated Axiom 0

$$\boxed{\text{Every physically realisable coherent system has finite coherence capacity.}}$$

## 6 Positive Stability Threshold

**Conditional Positive Stability Threshold Theorem.** If compact transport, finite coherence capacity, and return-closure hold, then every stable defect satisfies

$$m \geq m_{\min} > 0.$$

Within the PTG/PDT framework, this positive stability threshold is interpreted as the emergence of a positive mass gap.

### Clarification of Status

This is a conditional result:

If compact transport, finite coherence capacity, and return-closure hold physically,

then a positive stability threshold follows.

This is not a proof of the Yang–Mills mass gap in the conventional mathematical sense.

## 7 Primitive Mass Scale

PDT-II uses the electron as the present reference low-energy stable charged defect, yielding:

$$\frac{m_e}{m_0} = 72.$$

Axiom 0 ensures that this identification is physically meaningful:

- lighter charged configurations collapse,
- the electron is the lightest stable charged defect,
- it therefore serves as the natural reference configuration.

## 8 Discussion

PDT-I establishes the physical foundation of Phase Differential Theory. The key insights are:

- coherence has a finite capacity,
- curvature loading accumulates under transport,
- collapse occurs when loading exceeds capacity,
- return-closure is physically required,
- a positive stability threshold emerges.

These results prepare the ground for:

- the mass hierarchy (PDT-II),
- the coherence horizon and collapse dynamics (PDT-III).

## 9 Conclusion

PDT-I introduces Axiom 0 and shows that finite coherence capacity leads to a positive stability threshold interpreted as a positive mass gap within the PTG/PDT framework. PDT-I should therefore be viewed as introducing a derived physical principle and exploring its mathematical consequences within the PTG framework.