PTG-5: Defect Mass Scaling and Return-Closure Invariants

Graham Fincham
Delta Phi IP Ltd

Daniel Hilton
Independent Researcher

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Abstract

Phase Transport Geometry (PTG) interprets mass as the coherence burden required to sustain a stable compact defect on the phase manifold $(S^1)^6$. Earlier papers established the closure hierarchy (PTG-1), compact transport and curvature loading (PTG-2), the closure-compatible metric (PTG-3), and the sector structure and structural algebras (PTG-4). PTG-5 introduces the structural mass functional implied by these geometric and algebraic ingredients. Each factor in the mass expression is motivated by the preceding PTG papers, and the multiplicative structure reflects the independent contributions of closure, winding, loading, excitation, sector symmetry, and return closure. The return algebra of the triplet sector is $\mathrm{End}(\mathbb{C}^3)$, which has dimension 9, yielding the down-sector invariant $T_3 = 9$. This invariant plays a central role in the mass hierarchy developed in PDT-II. PTG-5 provides the structural bridge between the mathematical PTG framework and the physical interpretation developed in the PDT series.

This paper is part of the Phase Transport Geometry / Phase Differential Theory series, which develops a unified geometric and operator-theoretic framework for relational transport.

1 Introduction

PTG-1 through PTG-4 established the geometric and algebraic structure of Phase Transport Geometry:

- closure hierarchy and minimal manifold (PTG-1),
- compact transport and curvature loading (PTG-2),
- closure-compatible metric (PTG-3),
- sector structure and structural algebras (PTG-4).

PTG-5 introduces the structural mass functional implied by these ingredients. Each factor in the mass expression has been motivated independently in the earlier PTG papers. The multiplicative structure reflects the independence of the geometric and algebraic contributions to the coherence burden.

2 Primitive Mass Scale

The primitive mass scale is defined as

$$m_0 = \frac{2\pi\hbar}{c^2 T_0},$$

where T_0 is the primitive coherence interval.

This is the natural quantity with dimensions of mass that can be constructed from the primitive coherence interval together with $\hbar$ and c. The scale m_0 acquires its physical interpretation later in the PDT series.

PDT-II uses the electron as the reference stable charged defect for calibration, yielding

$$\frac{m_e}{m_0} = 72.$$

3 Baseline Mass Factors

Each defect is characterised by three baseline factors:

$$B_i = C_i W_i L_i.$$

Closure Factor

The closure factor counts the number of independent closure constraints in the sector:

$$C_1 = 1, \qquad C_2 = 2, \qquad C_3 = 3.$$

Winding Factor

The minimal nontrivial winding compatible with compact transport is

$$W_{\text{min}} = 9.$$

Loading Factor

The minimal curvature loading required for stability is

$$L_{\text{min}} = 4.$$

Baseline Values

Thus the minimal baselines are:

$$B_1 = 36, \qquad B_2 = 72, \qquad B_3 = 108.$$

4 Excitation Modes

PDT-MC-1 shows that only three excitation modes are stable:

$$g = 1, 2, 3.$$

The excitation contribution to mass is quadratic:

$$g^2.$$

5 Sector Symmetry Factors

Each defect belongs to one of the closure sectors:

$$S_1, \qquad S_2, \qquad S_3.$$

These factors encode the degeneracy of the defect under the structural algebras:

$$A(1) = \mathbb{C}, \qquad A(2) = \mathrm{End}(\mathbb{C}^2), \qquad A(3) = \mathrm{End}(\mathbb{C}^3).$$

6 Return-Closure Requirement

Under finite coherence capacity (PDT-I), residual phase cannot persist. A stable defect must therefore satisfy a return-closure condition:

$$T(y)v = v.$$

PDT-MC-1 shows that the return algebra of the triplet sector is

$$A(3) = \mathrm{End}(\mathbb{C}^3),$$

which has dimension

$$T_3 = 9.$$

Thus:

$$T_1 = 1, \qquad T_2 = 1, \qquad T_3 = 9.$$

7 Structural Mass Functional

Structural Mass Functional Theorem. Assuming:

- the closure hierarchy (PTG-1),
- compact transport (PTG-2),
- the closure-compatible metric (PTG-3),
- the structural sector algebras (PTG-4),
- and return closure (PDT-MC-1),

the PTG framework admits the structural mass functional

$$m_i = m_0 \, C_i W_i L_i \, g_i^2 \, S_i \, T_i,$$

with each factor representing an independent geometric or algebraic contribution to the coherence burden required to sustain a stable compact defect.

8 Corrected Baseline for Triplet Defects

The corrected baseline factor for triplet defects is

$$B_3^{\mathrm{tot}} = B_3 T_3 = 108 \times 9 = 972.$$

9 Interpretation

PTG interprets mass as the coherence burden required to sustain a stable compact defect. The structural mass functional reflects:

- closure structure,
- winding,
- curvature loading,
- excitation mode,
- sector symmetry,
- return-closure invariants.

The down-sector invariant $T_3 = 9$ is the dominant amplification factor for triplet defects and plays a central role in the mass hierarchy developed in PDT-II.

10 Conclusion

PTG-5 introduces the structural mass functional implied by the PTG framework. The return-closure requirement introduces the down-sector invariant $T_3 = 9$, leading to the corrected baseline factor for triplet defects. This structure provides the bridge between the mathematical PTG papers and the physical interpretation developed in the PDT series.