PTG-2: Compact Transport Geometry and Curvature Loading

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Abstract

Phase Transport Geometry (PTG) models coherent physical evolution using compact, periodic transport on the minimal phase manifold $(S^1)^6$ established in PTG-1. This paper introduces the compact transport potential

$$T(\theta) = K(1 - \cos\theta),$$

adopted as the minimal smooth periodic potential satisfying periodicity, symmetry, boundedness, and quadratic behaviour near equilibrium. The associated curvature operator distinguishes local spectral instability from global coherence loading. The separation between the spectral quantity $\lambda_{\max}(K)$ and the aggregate quantity $\mathrm{Tr}(K)$ is a defining structural feature of PTG dynamics. This distinction provides the structural motivation for introducing finite coherence capacity in PDT-I and underlies the mass-scaling and collapse dynamics developed in PTG-5, PDT-II, and PDT-III.

This paper is part of the Phase Transport Geometry / Phase Differential Theory series, which develops a unified geometric and operator-theoretic framework for relational transport.

1 Introduction

PTG-1 established the closure hierarchy (1,2,3) and the minimal phase manifold

$$M = (S^1)^6.$$

PTG-2 introduces the dynamical structure on this manifold. The central idea is that coherent evolution is governed by a compact transport potential. Unlike linear transport, compact transport ensures that phase differences remain bounded and periodic.

The curvature operator derived from this potential encodes both local and global stability properties. The distinction between local spectral instability and global coherence loading is one of the central mathematical contributions of this paper.

2 Compact Transport Potential

We adopt the compact transport potential

$$T(\theta) = K(1 - \cos\theta),$$

as the minimal smooth periodic potential satisfying:

- periodicity: $T(\theta + 2\pi) = T(\theta)$,

- symmetry: $T(-\theta) = T(\theta)$,
- boundedness: T is bounded above and below,
- quadratic behaviour near equilibrium:

$$T(\theta) = \tfrac{1}{2}K\theta^2 + O(\theta^4).$$

These properties make T the simplest admissible compact potential compatible with the closure structure of PTG-1. Any alternative smooth periodic potential would introduce unnecessary higher harmonics or break the minimality assumptions.

3 Curvature Operator

The curvature operator is the geometric Hessian of T with respect to the closure-compatible metric introduced in PTG-3:

$$K(\theta)X = \nabla_X(\nabla T)(\theta).$$

Pairwise coupling assumption

In this paper we assume pairwise phase-difference coupling, meaning that the transport potential depends only on differences $\theta_i - \theta_j$. Under this assumption, the Hessian takes the form

$$K_{ij} = K\cos(\theta_i - \theta_j).$$

4 Local Instability and Global Loading

Local instability is governed by the largest eigenvalue of K:

$$\lambda_{\max}(K).$$

Global coherence loading is governed by the trace:

$$C = \mathrm{Tr}(K).$$

These two quantities behave differently under transport and play distinct roles in PTG dynamics.

Local–Global Stability Theorem

For compact transport geometry, local instability and global coherence loading are distinct geometric quantities. Local instability is governed by the spectral quantity $\lambda_{\max}(K)$, whereas global coherence loading is governed by the aggregate quantity $\mathrm{Tr}(K)$.

Consequently, a configuration may remain spectrally stable while simultaneously exceeding the global coherence capacity.

Illustrative Example

Consider three phase directions with curvature contributions:

$$K = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 10 \end{pmatrix}.$$

Then:

$$\lambda_{\max}(K) = 10, \qquad \mathrm{Tr}(K) = 14.$$

A configuration may have all eigenvalues below a local instability threshold while the trace exceeds a global coherence capacity. This illustrates the structural separation between spectral and aggregate behaviour.

5 Local vs Global Instability

PTG distinguishes two fundamentally different types of instability:

- **Local instability:** a single mode becomes unstable when

$$\lambda_{\max}(K) > \lambda_{\mathrm{crit}}.$$

- **Global coherence collapse:** the total loading exceeds a capacity when

$$C = \mathrm{Tr}(K) > C_{\mathrm{crit}}.$$

This separation between spectral and aggregate behaviour provides the structural motivation for introducing finite coherence capacity in PDT-I.

6 Connection to Finite Coherence Capacity

The distinction between $\lambda_{\max}(K)$ and $\mathrm{Tr}(K)$ provides the structural motivation for the finite coherence capacity introduced formally in PDT-I:

$$\mathrm{Tr}(K) \leq C_{\max}.$$

A system may be locally stable (all eigenvalues below threshold) yet still collapse globally if the total loading becomes too large.

7 Discussion

PTG-2 provides the dynamical framework for the PTG/PDT programme. The compact transport potential ensures bounded evolution, while the curvature operator encodes both local and global stability properties.

The Local–Global Stability Theorem:

- clarifies the difference between local instability and global collapse,

- provides the structural motivation for finite coherence capacity (PDT-I),
- underlies the emergence of the mass gap,
- prepares the ground for the closure-compatible metric (PTG-3),
- supports the mass-scaling and collapse dynamics developed in PTG-5 and PDT-III.

8 Conclusion

PTG-2 introduces compact transport geometry and the curvature operator that governs coherent evolution on $(S^1)^6$. The separation between spectral instability and aggregate loading is a defining feature of PTG dynamics and forms the basis for the metric structure (PTG-3), sector organisation (PTG-4), mass scaling (PTG-5), and the physical interpretation developed in the PDT series.