PTG-4: Sector Structure, Chirality, and Structural Algebras

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Abstract

Phase Transport Geometry (PTG) organises the six phase degrees of freedom on the minimal phase manifold $(S^1)^6$ into triplet, doublet, and singlet closure sectors. These sectors support distinct classes of phase defects: collective closure defects in the triplet sector, orientation (chirality) defects in the doublet sector, and winding defects in the singlet sector. Each sector is canonically associated with a structural endomorphism algebra: $\mathrm{End}(\mathbb{C}^3)$, $\mathrm{End}(\mathbb{C}^2)$, and $\mathbb{C}$ respectively. These algebras classify the internal symmetry of the closure sectors and should not be interpreted as physical gauge groups. PTG-4 develops the geometric and algebraic structure of the closure sectors and states the Sector Algebra Theorem, proved in full in PDT-MC-1. This theorem provides the organisational backbone for mass scaling in PTG-5 and the physical interpretation developed in PDT-I and PDT-II.

This paper is part of the Phase Transport Geometry / Phase Differential Theory series, which develops a unified geometric and operator-theoretic framework for relational transport.

1 Introduction

PTG-1 established the closure hierarchy (1,2,3) and the minimal phase manifold

$$M = (S^1)^6.$$

PTG-2 introduced compact transport and curvature loading, and PTG-3 constructed the closure-compatible metric. PTG-4 now examines the internal structure of the closure sectors themselves.

The closure hierarchy induces a natural decomposition of the phase manifold into:

- a triplet sector supporting collective closure,
- a doublet sector supporting orientation (chirality),
- a singlet sector supporting integer winding.

These sectors behave differently under transport, curvature, and defect formation. Their associated structural algebras classify defect behaviour and provide the organisational framework for mass scaling.

2 Sector Decomposition

The six phase coordinates are grouped as follows:

$$(\theta_1, \theta_2, \theta_3) \in S^1 \times S^1 \times S^1, \qquad (\theta_4, \theta_5) \in S^1 \times S^1, \qquad \theta_6 \in S^1.$$

Triplet Sector

The triplet sector supports the primitive closure relation

$$1 + \omega + \omega^2 = 0.$$

This sector supports collective closure defects, where the three phases must return to a neutral configuration.

Doublet Sector

The doublet sector supports orientation (chirality). The two phases can be arranged in left- or right-handed configurations. Reversing the ordering corresponds to an orientation-reversing transformation (determinant -1), whereas preserving the ordering corresponds to an orientation- preserving transformation (determinant $+1$). This provides the geometric origin of chirality in PTG.

Singlet Sector

The singlet sector supports integer winding. It behaves like a standard S^1 degree of freedom with no closure redundancy.

3 Defect Classes

Each sector supports a distinct class of topological or geometric defects.

Triplet Defects

Triplet defects arise when the collective closure condition is frustrated. The triplet sector is canonically associated with the structural algebra

$$\mathrm{End}(\mathbb{C}^3),$$

which has dimension 9. This algebra classifies the internal symmetry of triplet defects and plays a central role in PTG-5 and PDT-II.

Doublet Defects

Doublet defects arise from orientation frustration. The associated structural algebra is

$$\mathrm{End}(\mathbb{C}^2),$$

which has dimension 4. These defects encode chirality and contribute to the structure of the mass-scaling factors in PTG-5.

Singlet Defects

Singlet defects correspond to integer winding numbers. Their associated algebra is

$$\mathbb{C}.$$

4 Structural Algebras

PTG does not assume gauge groups. The structural algebras should not be interpreted as physical gauge groups. Rather, they classify the internal symmetry of the closure sectors themselves.

The closure sectors are canonically associated with the endomorphism algebras:

$$\mathrm{End}(\mathbb{C}^3), \qquad \mathrm{End}(\mathbb{C}^2), \qquad \mathbb{C}.$$

These algebras:

- classify defect interactions,
- determine coherence loading contributions,
- shape the mass-scaling structure in PTG-5,
- provide the precursor to the physical interpretation in the PDT series.

Why these algebras?

The reduced phase spaces of the triplet, doublet, and singlet sectors have dimensions 3, 2, and 1 respectively. The maximal closure-preserving endomorphism algebra acting on each reduced space is the full endomorphism algebra:

$$\mathrm{End}(\mathbb{C}^n).$$

Any smaller algebra would fail to preserve all closure-compatible directions or would break sector symmetry. Thus the canonical structural algebras are uniquely determined by dimensionality and closure symmetry.

5 Sector Algebra Theorem

Sector Algebra Theorem. Under the closure hierarchy and closure-compatible metric, the return algebras of the triplet, doublet, and singlet sectors are

$$A(3) = \mathrm{End}(\mathbb{C}^3), \qquad A(2) = \mathrm{End}(\mathbb{C}^2), \qquad A(1) = \mathbb{C}.$$

The full proof is given in PDT-MC-1.

6 Discussion

PTG-4 reveals that the closure hierarchy induces a rich internal structure on $(S^1)^6$. The triplet, doublet, and singlet sectors support distinct defect types and are canonically associated with the structural algebras $\mathrm{End}(\mathbb{C}^3)$ and $\mathrm{End}(\mathbb{C}^2)$.

This sector structure is essential for:

- the primitive curvature scale (PTG-3),
- defect mass scaling (PTG-5),
- the down-sector invariant (PDT-II),
- the physical interpretation in PDT-I and PDT-II,
- the collapse dynamics developed in PDT-III.

7 Conclusion

PTG-4 develops the sector structure inherent in the closure hierarchy. The triplet, doublet, and singlet sectors support distinct defect classes and identify the structural algebras associated with the closure sectors. These algebras organise mass scaling and coherence behaviour in later PTG and PDT papers.